A Road-Adaptive Control Law for Semi-Active Suspensions

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This paper presents a road-adaptive control law for semi-active vehicle suspensions. In semiactive suspensions, damping coefficients are controlled so as to make the actual damper force as close to the desired damper force as possible at any time instance. The proposed control law consists of a road-adaptive sky-hook damping algorithm and a Road Detection Algorithm (RDA). This approach leads to the sprung mass and unsprung mass velocity feedback control law with time varying gains. The gains are tuned by the RDA. To evaluate the performance enhancement brought about by the proposed control law, the performance of a semi-active suspension with the proposed control law is compared to those of the sky-hook controlled semiactive suspension and a passive suspension. The controller has been implemented experimentally on a quarter car test rig and a semi-active damper with a 19 damping rates has been used to generate the desired semi-active force. The proposed control law provides adequate damping for the wheel hop frequency and improved performance compared to that of the sky-hook control law.

Key Words: Semi-Active Suspensions, Sky-hook Damping, Control, Road Adaptive, Disturbance, Ride Quality

1. Introduction

Active and semi-active suspensions for ground vehicles have been a very active subject of research for a long time due to their potential to improve vehicle performance (Karnopp, 1974, Yi, 1998). Many analytical and experimental studies performed recently concluded that active and semi-active suspensions can provide substantial performance improvements over optimized passive suspensions in general, and that semi-active suspensions can be nearly as effective as fully active suspensions in improving ride quality using state variable feedback. Semi-active suspensions were proposed in the early 1970's, showing that performance comparable to that of fully active suspensions can be achieved by the use of semiactive suspensions (Karnopp, 1974). It has been

well recognized that a semi-active suspension combines the advantages of both active and passive suspensions, i. e., it provides good performance compared to passive suspensions, and is economical, safe, and does not require either higher-power actuators or a large power supply. As semi-active damper hardware, microprocessors and micro-machined accelerometers become economical, semi-active suspensions have recently been commercialized on high performance automobiles (Konik, 1996).

Several semi-active suspension control laws have been proposed since the "clipped optimal" sky-hook damping control was suggested by Karnopp (1982). Cheok et al. (1985) proposed an "optimal model-following suspension". Cheok's approach guarantees optimality in minimizing the "model-following" error (i. e. the differences between a semi-active and the optimal active control) but not in the performance of the semi-active suspension itself. Kimbrough (1986) introduced a bilinear controller that guarantees improved performance over a passive suspension.

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Hrovat et al. (1988) proposed an approach for an optimal semi-active suspension. Tseng et al. (1991) developed an optimal control law with a time varying feedback gain matrix and a "steepest gradient" method and numerically compared the performance of alternative semi-active control laws. Novak (1996) introduced a new concept of semi-active control, the so-called ground-hook algorithm that consists of the sprung mass velocity feedback and tire deflection feedback. Hennecke and Zieglmeier proposed an adaptive damping strategy that selects constant damping on the basis of the frequency content of the road input and updates the damping rate when the road conditions change (Hennecke, 1988). The adaptive damping strategy can provide limited performance improvement since many roads have a broad frequency spectrum, and it has been recognized that superior performance can be achieved with rapidly varying dampers that can closely approach the behavior of active systems.

Despite alternative control laws the "clipped optimal" sky-hook control method is generally used since it is the best compromise between performance and practical implementation of semi-active suspensions.

In this paper, a "new road-adaptive sky-hook" control law is presented. The control law consists of a new adaptive sky-hook damping algorithm and a Road Detection Algorithm (RDA). This approach leads to the sprung mass and unsprung mass velocity feedback control law with time varying gains. The gains are tuned by the RDA. To evaluate the performance enhancement brought about by the proposed control law, the performance of a semi-active suspension with the proposed control law is compared to that of the sky-hook controlled semi-active suspension and with a passive suspension. The controllers are implemented experimentally on a quarter car test rig, and a semi-active damper with a 19 damping rates has been used to generate the desired semiactive force.

2. Semi-Active Suspensions

For automobiles with independent suspensions,



Fig. 1 A quarter car semi-active suspension model.

four-quarter-car models can be used to represent the entire car as far as designing active/semiactive suspensions for vibration isolation purposes is concerned. The quarter car model is shown in Fig. 1. A spring, a semi-active damper, sprung/unsprung masses, and a tire constitute the quarter car model. The tire is modeled as a spring and its damping is assumed to be negligible.

Define the following four states:

 $x_1 = z_s - z_u, x_2 = \dot{z}_s, x_3 = z_u - z_r, x_4 = \dot{z}_u$

Then the semi-active suspension is modeled as the following bilinear system:

$$\dot{x} = Ax + Bf_s + \Gamma \dot{z}_r$$
$$= Ax + Dxu + \Gamma w$$

where the variable force $f_s(=-u(\dot{z}_s-\dot{z}_u))$ is the damping force generated by the semi-active damper, the unknown disturbance w is the rate of change of road elevation, u is the controllable damping rate to be determined by a control strategy within a given range, and

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & 0 & -\frac{k_t}{m_u} & 0 \end{bmatrix},$$
$$D = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{m_s} & 0 & \frac{1}{m_s} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{m_u} & 0 & -\frac{1}{m_u} \end{bmatrix}$$
$$\Gamma = \begin{bmatrix} 0 & 0 & -1 & 0 \end{bmatrix}^T, \quad B = \begin{bmatrix} 0 & \frac{1}{m_s} & 0 & -\frac{1}{m_u} \end{bmatrix}^T$$

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In practice, real dampers have nonlinear forcevelocity characteristics, in which case the system is no longer bilinear. However the bilinear form is still useful from the controller/observer design point of view.

The damping coefficient is changed in real time so as to make $u(\dot{z}_s - \dot{z}_u)$ as close to the desired time-varying force $f_{s,des}$ as possible. The desired time varying force may be described in terms of full-state feedback:

$$f_{s,des} = -b_s(x_2 - x_4) + g_1x_1 + g_2x_2 + g_3x_3 + g_4x_4$$

where b_s is a passive damping coefficient and the
constant gains g_1 , g_2 , g_3 and g_4 can be chosen
using optimal control theory so as to minimize
the following cost functional:

$$J = \lim_{T \to \infty} \frac{1}{T} \mathbf{E} \left[\int_0^T (\dot{x}_2^2 + \rho_1 x_1^2 + \rho_2 x_2^2 + \rho_3 x_3^2 + \rho_4 x_4^2) dt \right]$$

The weighting factors ρ_1 , ρ_2 , ρ_3 and ρ_4 determine the relative importance of the states in minimizing the performance index. They may be chosen so as to emphasize improvement in ride quality or else to emphasize reduction in tire force variation.

A simple but effective control law for ride quality improvement (Butsuen, 1989) is to choose the desired force as

$$f_{s,des} = -b_s(x_2 - x_4) - b_{sky}x_2$$

This control law is well-known as the "skyhook" damping law (Karnopp, 1982) with b_{sky} the sky-hook damping coefficient. It attempts to attach a damper between the sprung mass and inertial space. It is shown that, as far as ride quality is concerned, sky-hook damping provides nearly as good a performance as full-state feedback does (Butsuen, 1989).

A simple tire deflection feedback control law is represented as follows:

$$f_{s,des} = -b_s(x_2 - x_4) + g_{opt}x_3$$

where g_{opt} is the optimal feedback gain. This control law atlempts to reduce dynamic tire loading (i. e. tire deflection) for road friendliness and performs better than sky-hook damping, especially at high frequencies as regards dynamic tire force reduction. In actuality, the variable damping rate u(t), is modulated in the following admissible space:

 $\Omega = \{ u \mid u_{\min} \leq u(\cdot) \leq u_{\max} \}$

A reasonable and very effective damping control law is obtained by changing within the admissible space so as to make the semi-active force f_s track the desired force $f_{s,des}$ as closely as possible:

$$u(t) = \begin{cases} u_{\min} & \text{if } u^*(t) \le u_{\min} \\ u^*(t) & \text{if } u_{\min} \le u^*(t) \le u_{\max} \\ u_{\max} & \text{if } u_{\max} \le u^*(t) \end{cases}$$

where

$$u^*(t) = -\frac{f_{s,des}}{(x_2 - x_4)}$$

3. A New Road-Adaptive Sky-Hook Control Law

A good semi-active suspension should provide high damping for low frequency inputs (i. e. $1 \sim 2$ Hz) to achieve good body isolation, low damping in the mid-frequency range $(3 \sim 7 \text{ Hz})$ for good comfort, adequate damping to control the wheel hop in the wheel hop frequency (approximately 10 Hz), and finally, increased damping in the high frequency range (i. e. above 15 Hz) for structural modes.

Although the sky-hook damping control provides good ride quality improvement, it does not provide adequate damping in the wheel hop frequency. Thus, dynamic tire force (i. e. tire force variations) is increased in the case of sky-hook damping control. The tire deflection feedback control law can provide good damping at wheel hop frequency, but implementation of the control law requires information on the tire deflection, and the tire deflection (x_3) is very difficult to measure.

In order to combine the advantages of both sky-hook damping and tire deflection feedback, we propose a new sky-hook control as follows:

$$f_{s,des} = -b_s(x_2 - x_4) - b_{sky}x_2 + b_{grox_4}$$

Since the sprung and unsprung mass accelerations \ddot{z}_s and \dot{z}_u can be measured by inexpensive accelerometers, and the sprung mass and unsprung mass velocities (x_2 and x_4) can be obtained with the acceleration measurements using filter and/or observer (Hedrick, 1994; Yi, 1998), the control law can be implemented with ease.

It was shown by Yi (1998) that the performance of a semi-active suspension with the new sky-hook control can be improved by using road-adaptive gains b_{sky} and b_{gro} which are adapted based on the road characteristics. The road-dependent optimal gains b_{sky} (Sky-Hook Gain) and b_{gro} (Ground-Hook Gain), which are chosen considering both the ride quality (i. e. the sprung mass accelerations) and the dynamic tire force (i. e. the tire deflections), are shown in Fig. 2. The optimal gains have been chosen via computer simulations by parametric studies. The " Frequency" in Fig. 2 indicates the frequency of



Fig. 2 Optimal road-adaptive gains.

the dominant harmonic or periodic excitations in road inputs that can be detected. Therefore, if the dominant frequency content of the road inputs is detected, then the desired semi-active force $f_{s,des}$ can be determined by a new "road-adaptive" sky-hook control law as follows:

$$f_{s,des} = -b_s(x_2 - x_4) - b_{sky}(f_{road}) x_2 + b_{gro}(f_{road}) x_4$$

where f_{road} is the dominant frequency content of the road inputs.

Figure 3 shows a block diagram describing a semi-active suspension system with the new roadadaptive sky-hook control law. The adaptive control algorithm, filters/observers and a Road Detection Algorithm constitute a semi-active controller. The filters/observers estimate suspension states such as the sprung mass velocity, the unsprung mass velocity and the suspension velocity based on the acceleration measurements. The RDA determines the optimal gains $b_{sky}(f_{road})$ and $b_{gro}(f_{road})$, on the basis of the frequency contents of the road input and updates the gains when the road conditions change. The RDA is described in detail in the next section. The controller computes the desired force $f_{s,des}$ based on the new adaptive sky-hook control law, determines the damping coefficient in real time such



Fig. 3 A semi-active suspension system with the new "road-adaptive" sky-hook control law.

that the damper force f_s is as close to the desired force as possible, and generates a control input for the semi-active damper. Vertical dynamics of the vehicle is considered, and control laws for heave motion control are investigated through quarter car simulations and quarter car rig tests.

4. Road Detection Algorithm

The RDA has been designed to adapt the optimal gains, $b_{sky}(f_{road})$ and $b_{gro}(f_{road})$, on the basis of the frequency contents of the road input. Figure 4 illustrates the RDA designed in this study. The RDA computes the optimal gains using acceleration measurements. Four band pass filters and a neural network constitute the RDA. Since the frequency content of the sprung mass accelerations is correlated with the road excitation, the frequency content of the road input can be detected by processing the acceleration measurements. Firstly, the output of the i-th band pass filter, $g_i(k)$, is computed using the acceleration measurements as follows:

$$g_i(k) = a_{i,1}g(k-1) + a_{i,2}g(k-2) + b_{i,1}y(k-1) + b_{i,2}y(k-2), \quad i=1, 2, 3, 4$$

where y(k) is the sprung mass acceleration measurement at time t = kT. T is a sampling time and $a_{i,j}$, $b_{i,j}$ are filter coefficients. Then, the normalized amplitude of the filter outputs, g_i (k), is used as inputs for the neural network. The outputs of the neural network are the optimal gains $b_{sky}(f_{road})$ and $b_{gro}(f_{road})$. The neural network consists of an input layer, a hidden layer with 5 nodes, and and output layer. The output, $y_1 (\equiv b_{sky})$ and $y_2 (\equiv b_{gro})$ are computed as follows:

$$b_{sky}(k) = y_1(k) = \sum_{i=1}^n w_{1,i} x_i(k) + b_1$$

$$b_{gro}(k) = y_2(k) = \sum_{i=1}^n w_{2,i} x_i(k) + b_2$$

where x_i , n, $w_{j,i}$ and b_j are the output from node i in the hidden layer, the number of the nodes in the hidden layer, the weight associated with the link connecting the output y_j and node i, and the threshold associated with the output y_j , respectively. The outputs of the nodes are computed as follows:

$$x_{i}(k) = \sum_{j=1}^{m} w_{j,i}A_{j,norm}(k) + t_{i}$$

$$A_{j,norm}(k) = \frac{rms(g_{j}(k))}{\max rms(g_{i}(k))}$$

$$rms(g_{j}(k)) = \sqrt{\frac{\sum_{l=k-l+1}^{k} g_{j}^{2}(l)}{I}}$$

where I is an integer, m, $w_{j,i}$ and t_j are the number of the input nodes, the weight associated with the link connecting the input g_j and node i, and the threshold associated with χ_{ij} respectively.

The weights and the thresholds were determined by the learning paradigm often referred to as backpropagation learning rule (Jang, 1997).

The performance of the RDA has been investigated for various road inputs. Measured road profiles have been used as the random road input, and road inputs with periodic excitations have been generated by adding sinusoidal road profiles to the measured road profiles. Figure 5 shows the

Road Detection Algorithm



Fig. 4 Road detection algorithm (RDA).



(a) Random road with 1 Hz periodic excitations



(b) Random road with 5 Hz periodic excitations



(c) Random road with 10 Hz periodic excitations



(d) Random road without periodic excitations Fig. 5 Sky-hook and ground-hook gains tuned by the RDA.

gains adapted by the RDA for road inputs with a periodic excitation. A sampling time of 5 milliseconds was used, and the last three-hundred outputs of the band pass filters have been used to compute the rms values (i. e. I = 300). The initial values for the sky-hook and ground-hook gains were set to be 4000 [Nsec/m] and 0 [Nsec/m], respectively, and the RDA started to adapt the gains at 1.5 seconds. Figure 5 (a) indicates that the sky-hook and ground-hook gains are tuned to approximately 4000 [Nsec/m] and 0 [Nsec/ m], respectively. It can be noted from figure Fig. 2 and Fig. 5 (a), (b), (c) and (d) that the gains are tuned in real time as designated. It should be noted that the response time is small and the gains are close to the optimal gains illustrated in Fig. 2.

5. Computer Simulation and **Experimental Results**

Experimental studies have been performed to evaluate the performance of a semi-active suspension with the proposed road-adaptive sky-hook control law. The semi-active suspension has been implemented on a quarter test rig. The implementation of the proposed control law needs sprung mass and unsprung mass absolute velocities to be available. The velocities were obtained from the sprung and unsprung mass acceleration measurements by a using second-order integrating filter which can remove the effects of D. C. offsets.

5.1 Quarter car test rig

The laboratory quarter car test rig is shown in



Fig. 6 Quarter car test rig.

Fig. 6. The quarter car test rig consists of a hydraulic power system, a road input generating system, a vehicle dynamics simulating system, sensors, and an electronic control system. The parameter values for the quarter car test rig are given in Table 1. A semi-active damper with 19 states has been used to generate the desired semi-active force. The semi-active damper is controlled by a 2 phase PWM drive type step motor, and a Pentium personal computer has been used as the controller for the semi-active suspension.

The damper force versus velocity curves for the semi-active damper are shown in Fig. 7. Figure 7 illustrates that different forces can be generated by the semi-active damper for the same damper velocity. The damper characteristics were treated as piecewise linear, and a look-up table was used to determine the appropriate damper state.

5.2 Simulation and experimental results

A comparison of the desired and sctual damper forces for 1.1 Hz sinusoidal road inputs are shown in Fig. 8. The frequency is close to the natural frequency of the sprung mass. The desired force, $f_{s,des}$, was determined by the proposed



Fig. 7 Force-velocity curves for the semi-active damper.







control law, and the damper state was controlled so as to make the damper force f_s track the desired force as close as possible. The tracking performance is good when the passivity constraint (Butsuen, 1989) is satisfied and the actual damper



(b) Simulation results



force is kept at a minimum when the passivity constraint is not satisfied, i.e. when a power supply is necessary. As noted from the figures, the simulation predicts experimental results quite accurately.

A comparison of the sprung accelerations is shown in Fig. 9. "Semi-active" and "Passive" indicate the semi-active suspension with a multistate damper and a suspension with an optimal passive damper, respectively. The semi-active and passive dampers used in the experiments are for a full-size passenger car, and the characteristics of the passive dampers have been designed by a damper manufacturer to optimize the performance of the passive suspensions. The tire used in the full-size passenger car, the spring with the same spring constant as the front suspension of the car, and an equivalent sprung mass measured from the car have been used for the quarter car test rig. The quarter car rig has been designed such that the dynamic response of the quarter car is as close to that of the full size passenger car as



(b) Simulation results

Fig. 10 Comparisons of the dynamic tire forces for the passive and semi-active suspensions.

possible. A reduction in acceleration is obtained by the semi-active suspension.

Figure 10 shows a comparison of the dynamic tire forces of the semi-active and passive suspensions. The force is measured by a load cell. A reduction in the dynamic tire force can be obtained in the sprung mass mode (typically 0.5-3Hz) frequency range by the semi-active suspension.

5.3 Comparisons between sky-hook control and the road-adaptive sky-hook control

The frequency characteristics of the sky-hook control and the new adaptive sky-hook control are compared to those of the passive suspension. Since the semi-active suspension system is not linear, the frequency response is not uniquely defined. In addition, since the damper force -velocity curve of the passive damper is nonlinear, the passive suspension is also not linear. In this study, the magnitude of the frequency response at the desired frequencies was obtained



(a) The dynamic tire forces

Fig. 11 Comparisons between the sky-hook and the new adaptive sky-hook control laws.

as the ratio of the root mean squared (rms) values of the measurements and the road velocity inputs at those frequencies.

Figure 11 shows the comparison of the frequency response characteristics of the semi-active suspensions. The characteristics for the optimal passive suspension is also provided for reference. It is illustrated that a significant reduction of the sprung mass acceleration is obtained by the semiactive suspensions in the frequency range 0.5-7Hz. The accelerations of the semi-active suspensions are similar to that of the passive suspension at the wheel hop mode. This is due to the invariant property of the vehicle suspension. As mentioned in Sec. 3, the sky-hook damping control does not provide adequate damping in the wheel hop frequency and it can be seen from Fig. 11 (b) that, in the case of sky-hook control, the dynamic tire force is increased compared to that of the passive suspension. The new adaptive skyhook control provides good properties by adapting the gains in the following manner: i) high damping for low frequency inputs (1-2Hz); ii) low damping in the mid-frequency ranges $(3 \sim 7)$ Hz); iii) adequate damping for the wheel hop frequency (approximately 10 Hz). It is illustrated that the new adaptive sky-hook control provides improved performance compared to that of the sky-hook control.

6. Conclusions

A road-adaptive sky-hook control law for semi-active suspensions was presented. The proposed control law consists of a new sky-hook control law with road-adaptive gains and a Road Detection Algorithm (RDA). The new sky-hook control law is a combination of the sprung mass and unsprung mass velocity feedback with time varying gains, and the unsprung mass velocity feedback is seen to be excellent in increasing damping at the wheel hop frequency. The RDA has been designed to adapt the optimal gains on the basis of the frequency content of the road inputs. Four band pass filters and a neural network constitute the RDA.

The performanae of the proposed controller was verified via experimental studies using a quarter-car test rig. The proposed control law provides adequate damping for the wheel hop frequency and improved performance compared to that of the sky-hook control law. Consequently, a performance enhancement in ride comfort and road holding is obtained by the proposed control law.

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